

The Influence of Markov Chain and Properties of Principal Component Solutions in the Analysis of Share Price Movements for Stock Market

Dagogo Allen Wokoma BISHOP¹, Innocent Uchenna AMADI², George ISOBEYE³

^{1,2}Department of Mathematics/Statistics, Captain Elechi Amadi Polytechnic, Rumuola, Port Harcourt, Nigeria

³Department of Mathematics & Statistics, Ignatius Ajuru University of Education, Rumuolumeni, Port Harcourt, Nigeria

Article Info:

Article History:

Received: 2024-05-20

Revised: 2024-06-14

Accepted: 2024-07-08

Keyword:

Stochastic Analysis, Markov Chain, Capitalization, Transition Matrix

Corresponding Author:

Dagogo Allen Wokoma Bishop

Email:

dagywoksy@gmail.com

Paper Type:

Research Paper



Abstract:

Purpose:

Stock market performance and operation have been widely recognized as a viable investment field in financial markets. Therefore, this paper studied the stochastic analysis of the Markov chain and PCA in the closing share price data of Access and Fidelity banks (2016-2022) through the Nigeria Stock Exchange. The share prices were transformed into a 3-step transition probability matrix solution to cover this number of years.

Methodology:

This research uses the Markov chain method, which defines a stochastic process. Mathematically, a stochastic process can be defined as a collection of random variables ordered in time and determined at a series of time points that may be continuous or discrete.

Findings:

The criteria for obtaining four share prices formed from the two merged banks' 2x2 matrices were given, and analytical solutions of principal components were considered for future stock price changes.

Implication:

The solution matrix of the two merged banks showed that they have the best probability of price increasing shortly: 12%, the best probability of reducing in the future by 22%, and the best probability of no-change shortly by 21%, which is a tool for proper decision making in the day-to-day management of the bank; which shows it is profit making organization and are hopeful for future investment plans both short or long term respectively.

INTRODUCTION

The vital decision investors or owners of corporations will make is how to allocate funds to every business unit properly. It is due to unpredictability in stock market prices. Therefore, the above constraints need a viable mathematical model that guides investors' decision-making. So, in an investment, decision-making is one of the significant gears that positively or negatively affect investors. When good decisions are made, the financial strength of the investments will increase, but when incorrect decisions are made, the businesses begin to smash down.

Many researchers have written widely on demonstrating stock prices using the Markov chain, and results are obtained in numerous ways, just as in the work of (Agwuegbo et al., 2010). Examined stock market prices due to their fluctuations and influences on a country's financial lives and economic health. Their findings showed that stock price is a random work and no investor can alter the fairness and unfairness of a stock price as defined by expectation (Lakshmi & Jyothi, 2020). Studied Markov process of stock market performance. In their result, oil India exhibits a higher chance of stability with no significant increase or decrease (Davou, 2013). Studied Markov chain model on share price movement. Results showed that GT Bank shares more change hands than FB Bank (Agbam & Udo, 2020).

Considered the Markov chain model on forecasting of stock prices in Nigeria. Their result Markov chain model was determined based on the probability transition matrix (Amadi et al., 2022). Explored the stochastic analyses of the Markov chain in finite states. Their work replicated the use of a 3-state transition probability matrix, which enabled them to proffer precise conditions for obtaining the expected mean rate of return of each stock (Mettle et al., 2014). Considered stochastic analysis of share prices. Results showed the precise conditions for determining the expected mean return time for stock price, improving investment decisions based on the highest transition probabilities.

Similarly, Amadi et al. (2022) studied the stock market behavior using the Markov chain. The study reveals that regardless of the bank's current share price, steady-state probabilities of share price remain the same

throughout the iteration (Adeosun et al., 2015). Introduced a Markov chain model for stock market trend forecasting. The study revealed that the Markov chain model was more effective in analyzing and predicting the stock market index and closing stock price under the market mechanism (Davies et al., 2019). Studied long-run prospects of security prices in Nigeria, where the data were collected from randomly selected banks from Nigeria's banking sector. The analysis suggested that the price levels of Nigerian banks were likely to remain relatively stable in the long run, irrespective of the current situation (Ofomata et al., 2017). Examined the long-run behavior of eight Nigerian banks' closing price of shares using the Markov chain model. They computed a limiting distribution transition probability matrix of the share price and found that despite the current situation in the market, there is hope for Nigerian bank stocks. It was concluded that the results derived from the study will be helpful to investors. See for considerable extensions in this area of study (Dmouj, 2006; Ugbebor et al., 2001; Osu et al., 2009; Bairagi & Kakaty, 2015; Zhang & Zhang, 2009; Eseoghene, 2011; Christain & Timothy, 2014).

The prediction of possible states of the share price is more complicated due to the inherent stochastic behavior of prices, which rises up and down, making lives uneasy. Therefore, the share price data of Access and Fidelity banks were used to understand their price movements and their future merging effectively. The above concepts need stochastic analysis of the Markov chain to follow up realistically share price movement characterized by volatility. The model presented in this study will give Fidelity and Access clear directions when the two banks merge in decision-making concerning share prices, as it helps them understand the dynamics and patterns of the share prices that would be followed for optimum trading and profit. Secondly, it will account for the level of the proportion of the first PC of future price changes.

This paper aims to study the effect of the Markov chain in finite states with Principal Component Analysis (PCA) in the analysis of share prices of two merged banks. This paper extends the work of Osu et al. (2019) by incorporating PCA to study the variations in the two merged banks.

This paper is arranged as follows: Section 2.1 presents the materials and method, Results and Discussion are presented in Section 3.1, and the paper is concluded in Section 4.1.

METHODS

The purpose of this paper on the Markov chain is to start by defining the stochastic process. It can also be seen as a statistical event that evolves following probabilistic laws. A stochastic process may be mathematically defined as a collection of random variables ordered in time and defined at a set of time points, which may be continuous or discrete. Because a stochastic process is a relation of random variables, its requirement is similar to that for random vectors.

Definition 1: A stochastic process is said to be a Markov chain if the Markov property is satisfied:

$$P(X_{n+1} = j / X_0, X_1, \dots, X_n) = P(X_{n+1} = j / X_n)$$

For all $n \geq 0$ and $i, j \in S$ (state space)

Knowing that the Markov property given (1) is equivalent to the ease of the following for each $j \in S$ is sufficient.

$$P(X_{n+1} = j / X_{n1}, X_{n2}, \dots, X_{nk}) = P(X_{n+1} = j / X_{nk})$$

(for any $n_1 < n_2 < \dots, n_k \leq n$)

Assuming $X_n = i$ implies that the chain is in the i th state at the n th step. It can also be said that the chain has the value i or 'being in state i '. The idea behind the chain is described by its transition probabilities:

$$P(X_{n+1} = j / X_n = i)$$

They are dependent on i, j and n .

Definition 2: The chain is said to be homogeneous if the following are stated below;

$$P(X_{n+1} = j / X_n = i) = P(X_1 = j / X_0 = i)$$

For all n, i, j .

The transition matrix $P = (P_{ij})$ $n \times n$ is a matrix of transition probabilities.

$$P_{ij} = P(X_{n+1} = j / X_n = i)$$

Hence, the transition probabilities with a homogenous Markov chain are always stationary at a point.

Theorem 3: Suppose P is a stochastic matrix, which implies the following:

i) Has non-negative entries or $P_{ij} \geq 0$ (ii)

$$\sum_j P_{ij} = \sum_j P(X_{n+1} = j / X_n = i) = \sum_j P(X_1 = j / X_0 = i)$$

Please clarify whether you are referring to stationarity or point of convergence.

Proof: (i) Each associated entry in P is a transition probability P_{ij} and is a probability $P_{ij} \geq 0$.

$$(ii) \sum_j P_{ij} = \sum_j P(X_{n+1} = j / X_n = i) = \sum_j P(X_1 = j / X_0 = i)$$

Which is stationarity.

$$P(X_i \in S / X_0 = i) = 1.$$

Theorem 4 : (Chapman-Kolmogorov Equations).

$$P_{ij(m+n)} = \sum_{r=1}^n P_{ir(m)} P_{rj(n)} \text{ Since } P_{m+n} = P_m P_n \text{ and so on } P_n = P^n, \text{ the } nth \text{ power of } P.$$

$$P_{ij(m+n)} = P(X_{m+n} = j / X_0 = i)$$

$$\text{Proof: } \sum_r P(X_{m+n} = j, X_m = r / X_0 = i) \\ \sum_r P(X_{m+n} = j / X_m = r / X_0 = i) P(X_m = r / X_0 = i)$$

Using the following probability rule:

$$P(A \cap B / C) = P(A / B \cap C) P(B / C) \text{ and setting } A = \{X_{m+n} = j\}, B = \{X_m = r\}, \text{ and } C = \{X_0 = i\}$$

Using Markov property yields

$$P_{ij(m+n)} = \sum_r P(X_{m+n} = j / X_m = r) P(X_m = r / X_0 = i) \\ \sum_r P_{rj(n)} P_{ir(m)} \\ \sum_r P_{ir(m)} P_{rj(n)}$$

Hence $P_{m+n} = P_m P_n$ and so $P_n = P^n$, the power of P .

To obtain an estimate of the transition probability as follows

$$P_{ij} = P(X_t = j / X_{t-1} = i), \text{ for } j = 0, 1, 2, 3, \dots, N$$

$$P_{ij} = \begin{cases} P & \text{if } j = 1 + i \\ q = 1 - P & \text{if } j = i - 1 \\ 0 & \text{otherwise} \end{cases}$$

Where $k + 1$ is the number of states?

$$\left. \begin{aligned} n_{ij} &= \sum_{i=1}^n P_{ij} \text{ for } i = 0, 1, 2, 3 \\ \frac{n_{ij}}{n_i} &\text{ for } i = 0, 1, \dots, k \end{aligned} \right\}$$

However, $k = 3$ it is an estimate of the transition matrix.

$$\hat{P}_{ij}(\text{FIDELITY})_{2016-2022} = \begin{pmatrix} \hat{P}_{00} & \hat{P}_{01} & \hat{P}_{03} \\ \hat{P}_{10} & \hat{P}_{11} & \hat{P}_{12} \\ \hat{P}_{20} & \hat{P}_{21} & \hat{P}_{22} \end{pmatrix}$$

$$\hat{P}_{ij}(\text{ACCESS})_{2016-2022} = \begin{pmatrix} \hat{P}_{00} & \hat{P}_{01} & \hat{P}_{03} \\ \hat{P}_{10} & \hat{P}_{11} & \hat{P}_{12} \\ \hat{P}_{20} & \hat{P}_{21} & \hat{P}_{22} \end{pmatrix}$$

$$\hat{P}_{ij}(\text{ACCESS-FIDELITY})_{2016-2022} = \begin{pmatrix} \hat{P}_{00} & \hat{P}_{01} & \hat{P}_{03} \\ \hat{P}_{10} & \hat{P}_{11} & \hat{P}_{12} \\ \hat{P}_{20} & \hat{P}_{21} & \hat{P}_{22} \end{pmatrix}$$

Setting $i = j = 0, 1, 2$ for $k = 3$

Developing Markov Chain Model of Access Fidelity banks Share Prices. For proper accuracy of the Markov chain model for future events, it must be developed to predict share price movement. The initial share prices need to be in three finite states as follows:

R: represents the probability of share price reducing shortly, I: represents the probability of share price increasing shortly, NO-change: represents the probability of share price not changing shortly

However, the probability of the transition matrix shows the proper explanation of the Markov chain. Every element in the matrix communicates. In order to form three states of the Markov process, we need to have the following table below:

Table 1. Transition Probability Matrix

State	1	2	3	Total of Row
1	P_{11}	P_{12}	P_{13}	T_1
2	P_{21}	P_{22}	P_{23}	T_2
3	P_{31}	P_{32}	P_{33}	T_3

Each entry P_{ij} indicates how often a transition is made from one state i to state j . The transition matrix is computed by dividing every element in each row through the total. Nevertheless, this project studies Fidelity share price data collected (Osu et al., 2019).

Principal Component Analysis of the Share Price Variables. Definition 3: According to Udom (2015), suppose $\underline{X}$ has a joint distribution that has a variance matrix Σ with eigenvalues $\lambda_1, \lambda_2 \geq \dots \geq \lambda_p \geq 0$, considering the random variables $y_1, \dots, y_p$, which are linear combinations of the $X_1, \dots, X_p$, i.e.:

$$\left. \begin{aligned} y_1 &= \underline{l}_1' \underline{X} = l_{11}X_1 + \dots + l_{p1}X_p \\ &\vdots \\ y_p &= \underline{l}_p' \underline{X} = l_{1p}X_1 + \dots + l_{pp}X_p \end{aligned} \right\}$$

The y_i 's will be PC if uncorrelated, and the variances y_1, y_2 are as significant as possible. Recall that $y_i = \underline{l}_i' \underline{X}$ to look at the amount of information that is y_i . We can consider the proportion of the total population variance due y_i .

$$\frac{\lambda_i}{\lambda_1 + \lambda_2 + \dots + \lambda_p}, i = 1, \dots, p$$

Hopefully, the proportion is large for, e.g., $i = 1, 2, 3$.

RESULTS AND DISCUSSION

The data for this paper is obtained from the work (Osu et al., 2019). The closing share market price performances of two banks amalgamated in limited circumstances should be shown. The share price covers 2016-2022 and is retrievable from the Nigeria Stock Exchange (NSE).

Table 2. Share price of Fidelity Bank, PLC from 2016-2022

Share price movements	Reducing(R) is Share Price	Increasing(I) it's share price	No(N) change in price	Row totals
R	415	62	138	615
I	61	121	81	263
N	139	80	384	603

Table 3. Share price of Access Bank, PLC from 2016-2022

Share price movements	Reducing(R) is Share Price	Increasing(I) it's share price	No(N) change in price	Row totals
R	410	80	126	616
I	79	98	92	269
N	127	91	378	596

Table 4. Share price of two Banks merged from 2016-2022

Share price movements	Reducing(R) is Share Price	Increasing(I) it's share price	No(N) change in price	Row totals
R	825	142	264	1231
I	140	219	173	532
N	266	171	762	1199

The transition probability matrix for Access Fidelity banks merged according to (Osu et al., 2019).

$$ACCESS - FIDELITY = \begin{pmatrix} 0.6702 & 0.1154 & 0.2145 \\ 0.2632 & 0.4117 & 0.3252 \\ 0.2219 & 0.1426 & 0.6355 \end{pmatrix}$$

Minimum share price criteria: matrix (Amadi et al., 2022).

$$ACCESS - FIDELITY = \begin{pmatrix} 0.1154 & 0.2145 \\ 0.2219 & 0.1426 \end{pmatrix}$$

We formed the matrix from the estimates of the probability transition matrix of two banks merged, which information on the share price movements will account for the total proportion variability in the share price.

Table 5. The transition Probability Matrix of two merged Banks Share Market Prices with Means, Standard deviations, Kurtosis, and Skewness

a_{11}	a_{12}	a_{13}	Mean	Std	Kurtosis	Skewness
0.6702	0.1154	0.2145	0.3334	0.2959	1.500	0.6189
0.2632	0.4117	0.3252	0.3334	0.0746	1.500	0.1987
0.2219	0.1426	0.6355	0.3333	0.2647	1.500	0.6364

In Table 5, the mean indicates the probability of share price changes on average of Access Bank, which shows 0.3333 throughout investments; see column 4. The standard deviations in column 5 indicate that volatility always affects different price changes. Then, kurtosis measures the tailedness of entire share prices. Finally, the share price skewness measures the level of distortion in the data set, which guides an investor based on decision-making.

Table 6. Variations of future share prices of two merged banks according to the trading days

Trading Days	0.1154	0.2145	0.2219	0.1426
2	0.2308	0.429	0.4438	0.2852
4	0.4616	0.858	0.8876	0.5704
6	0.6924	1.287	1.3314	0.8556
8	0.9232	1.716	1.7752	1.1408
10	1.154	2.145	2.219	1.426
12	1.3848	2.574	2.6628	1.7112
14	1.6156	3.003	3.1066	1.9964
16	1.8464	3.432	3.5504	2.2816
18	2.0772	3.861	3.9942	2.5668
20	2.308	4.29	4.438	2.852

In Table 6, a little increase in the number of trading days also increases the future share prices of the two merged banks in their operations. It also implies that profit increases over time, giving business cycles more flexibility in adapting to market demands. The benefit of this assessment is to avert severe depletion of capital investments, which may endanger profit-making throughout the trading period of the capital investments.

However, the above valuations of the Fidelity and Access banks offer an eye opener of this stochastic analysis that will enhance investment decisions. The entire entry stipulates price changes for short and long-term business plans.

Principle Component Analysis of Two Banks Merged Share Price Movement Variations.

$$ACCESS - FIDELITY = \begin{pmatrix} 0.1154 & 0.2145 \\ 0.2219 & 0.1426 \end{pmatrix}, ACCESS - FIDELITY - \lambda I = 0$$

Solving the above share price matrix gives:

$$\lambda_1 = -0.0896, \lambda_2 = 0.3476$$

Solving, we have the following systems of equations.

$$0.205K_1 + 0.2145K_2 = 0$$

$$0.2219K_1 + 0.2322K_2 = 0$$

From (4.1) $0.205K_1 = -0.2145K_2$, $K_2 = \frac{0.205}{0.2145} = -0.9557$ putting K_2 in (4.2) gives

$$0.2219K_1 + 0.2322(-0.9557) = 0, 0.2219K_1 = 0.22191354 = K_1 = \frac{0.22191354}{0.2219} = 1.0001$$

Any vector of the form says form:

$$K_1 = \begin{pmatrix} 1.0001 \\ -0.9557 \end{pmatrix} = \begin{pmatrix} 1.0001C \\ -0.9557C \end{pmatrix}; \text{ say is an eigenvector corresponding to } \lambda_1 = -0.0896$$

$$-0.2322K_1 + 0.2145K_2 = 0$$

$$0.2219K_1 - 0.205K_2 = 0$$

From (4.3) $-0.2322K_1 = -0.2244K_2$, $K_2 = \frac{0.2322}{0.2145} = 1.0825$, putting K_2 in (4.4) gives

$$0.2219K_1 - 0.205(1.0825) = 0, 0.2219K_1 = 0.2219125 = 0, K_1 = \frac{0.2219125}{0.2219} = 1.0001$$

Any vector of the form says form:

$$K_2 = \begin{pmatrix} 1.0001 \\ 1.0825 \end{pmatrix} = \begin{pmatrix} 1.0001C \\ 1.0825C \end{pmatrix}; \text{ say is an eigenvector corresponding to } \lambda_2 = 0.3476$$

To obtain normalized eigenvectors for the share price of Access Bank:

$$K_1'K_1 = 1, (1.0001C \ -0.9557C) \begin{pmatrix} 1.0001C \\ -0.9557C \end{pmatrix}, 1.00020001C^2 + 0.912025C^2 = 1, 1.91222501C^2 = 1$$

$$C^2 = \frac{1}{1.91222501}, C = \frac{1}{\sqrt{1.91222501}}, e_1 = \begin{pmatrix} \frac{1.0001}{\sqrt{1.91222501}} \\ -0.9557 \\ \frac{1}{\sqrt{1.91222501}} \end{pmatrix}, \begin{pmatrix} 0.7232 \\ -0.6911 \end{pmatrix}$$

$$K_2'K_2 = 1, (1.0001C \ 1.0825C) \begin{pmatrix} 1.0001C \\ 1.0825C \end{pmatrix}, 1.00020001C^2 + 1.17180625C^2 = 1, 2.17200626C^2 = 1$$

$$C^2 = \frac{1}{2.17200626}, C = \frac{1}{\sqrt{2.17200626}}, e_1 = \begin{pmatrix} -1.0001 \\ \sqrt{2.17200626} \\ 1.0825 \\ \sqrt{2.17200626} \end{pmatrix}, \begin{pmatrix} -0.7232 \\ 0.7345 \end{pmatrix}$$

$$Y_1 = e_1'K = 0.7232K_1 - 0.6911K_2$$

$$Y_2 = e_2'K = 0.7345K_1 + 0.7345K_2$$

To calculate the principal component of Access bank share price accounted for the 1st PC

$$\lambda_1 = -0.0896, \lambda_2 = 0.3476, \frac{\lambda_1}{\lambda_1 + \lambda_2} = 0.5 = 50\%$$

Two eigenvalues represent the total Fidelity share price variance the principal component can explain. The represents the losses made all through the trading days by Fidelity Bank PLC. So, it is greater than zero, which is a good sign of a high investment return on the side of Fidelity Bank, whose aim and passion is to maximize profit. More so, the eigenvectors determine the share price direction in terms of changes in the short-run and long-run, respectively. The shows the average returns of two banks merged in the future.

The two banks merged, Access and Fidelity (2017-2021), Have a 67% of reducing their price, a 12% chance of increasing their price shortly, and a 21% chance of no change in price. Also, in the same circumstances, there is a 26% chance of reducing its price, a 41% chance of increasing its price, and a 33% chance of no change in price.

Finally, there is a 22% chance of reducing its price, a 14% chance of increasing its price, and a 64% Chance of no change in price. Overall, the overall predicted prices for the above companies give a 22% chance of reducing their price, a 14% chance of increasing their price, and a 64% chance of no change in price.

CONCLUSION

Markov chain is a mathematical tool used in modeling the share price movement of stochastic processes since each finite state communicates for proper management decision-making. Therefore, this project studied the stochastic analysis of the Markov chain and PCA in Access's closing share price data (2016-2022) via the Nigeria Stock Exchange. The share prices were transformed into a three-step transition probability matrix solution to cover this number of years. The future share price changes were known. The criteria for obtaining four share prices, which formed a 2x2 matrix, were given from two merged banks, and analytical solutions of principal components were considered for future stock price changes. The solution matrix of the stochastic analysis showed that Access Bank, PLC has the best probability of price increasing shortly: 12%, the best probability of reducing in the future by 21%, and the best probability of no-change shortly by 20%, which is a tool for proper decision making in the day-to-day management of the bank. More so, From the Fidelity Bank, PLC has the best probability of price increasing shortly: 10%, the best probability of reducing in the future by 23%, and the best probability of no-change shortly by 22%, which is a tool for proper decision making in the day-to-day management of the bank. Finally, the two banks merged; has the best probability of price increasing shortly: 12%, the best probability of reducing in the future by 67%, and the best probability of no-change shortly by 21%, which is a tool for proper decision making in the day-to-day management of the bank. The PC analysis shows the average returns of the two banks that merged shortly.

However, this study investigates three states of the transition matrix with PCA. The stochastic differential equation problem is an exciting area of further study.

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